

Geometric measure of quantum discord for a two-parameter class of states in a qubit-qutrit system under various dissipative channels*

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Quantum discord, as a measure of all quantum correlations, has been proposed as the key resource in certain quantum communication tasks and quantum computational models without containing much entanglement. Dakić, Vedral, and Brukner [Phys. Rev. Lett. **105**, 190502 (2010)] introduced a geometric measure of quantum discord (GMQD) and derived an explicit formula for any two-qubit state. Luo and Fu [Phys. Rev. A **82**, 034302 (2010)] introduced another form of GMQD and derived an explicit formula for arbitrary state in a bipartite quantum system. However, the explicit analytical expression for any bipartite system was not given. In this work, we give out the explicit analytical expressions of the GMQD for a two-parameter class of states in a qubit-qutrit system and study its dynamics for the states under various dissipative channels in the first time. Our results show that all these dynamic evolutions do not lead to a sudden vanishing of GMQD. Quantum correlations vanish at an asymptotic time for local or multi-local dephasing, phase-flip, and depolarizing noise channels. However, it does not disappear even though $t \rightarrow \infty$ for local trit-flip and local trit-phase-flip channels. Our results maybe provide some important information for the application of GMQD in hybrid qubit-qutrit systems in quantum information.

Keywords: quantum correlations, geometric measure of quantum discord, dynamics, a qubit-qutrit system, a two-parameter class of states

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1. Introduction

Quantum systems with quantum correlations, have some fundamental applications for quantum information processing. However, entanglement does not describe all the aspects of the quantum correlations exhibited by a multipartite quantum system. There are some other kinds of quantum correlations without entanglement that are also responsible for the quantum advantages over their classical counterparts [1–10]. Aiming at capturing such correlations contained in a bipartite quantum state, Ollivier and Zurek [11] introduced quantum discord as a measure of all quantum correlations in 2001. Recently, quantum discord attracted a lot of attention. Unfortunately, quantum discord is difficult to calculate and it is usually based on a numerical maximization procedure. There are few analytical expressions, including some special cases [12–16]. For general two-qubit mixed states, the situation is more complicated. To avoid this difficulty, Dakić *et al.* [17] introduced a geometric measure of quantum discord (GMQD) which measures the quantum correlations of a quantum system in a given state through the minimal Hilbert-Schmidt distance between the given state and a zero-discord state, and they derived an explicit formula for evaluating the GMQD for any two-qubit state in 2010. Subsequently, Luo and Fu [18] introduced another form of GMQD and derived an explicit formula for evaluating the GMQD for an arbitrary state in a bipartite quantum system. However, using this way, usually, it is also difficult to calculate GMQD, and its explicit analytical expression for any bipartite system was not given.

Besides the characterization and quantification of quantum correlations, another important problem is the behavior of these correlations under the action of decoherence. Recent results of the dynamics of the quantum correlation for a certain class of states in a two-qubit system under the influence of common noise channels show that the quantum correlations vanishes at asymptotic time. It maybe include a peculiar sudden change in behavior and is more resistant to the action of the environment than entanglement [19–24]. Quantum correlations for multi-valued quantum system is a new and immature research area. Ali [16] studied the quantum discord for a two-parameter class of states in a $2 \otimes d$ system in 2010. For a qubit-qutrit ($2 \otimes 3$) system, Ann [25] and Kan [26] studied its entanglement dynamics under the influence of dephasing and depolarizing channels, respectively. Ali *et al.* [27, 28] and Ramzan *et al.* [29] studied its entanglement dynamics under phase damping and amplitude damping channels. Karpat *et al.* investigated its correlation dynamics under dephasing in 2011 [30]. All the states concerned in previous works are some classes of

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the states in a qubit-qutrit system.

In this paper, we devote to investigate the GMQD for a two-parameter class of states in a qubit-qutrit system and the dynamics of GMQD under the influence of various dissipative channels [i.e., both two independent (multi-local) and only one (local) dephasing, phase-flip, bit- (trit-) flip, bit- (trit-) phase-flip, and depolarizing channels] in the first time. Analytical results are presented. Our results show that all these dynamic evolutions do not lead to a sudden vanishing of GMQD. Quantum correlations vanish at an asymptotic time for local or multi-local dephasing, phase-flip, and depolarizing noise channels. However, it does not disappear even though $t \rightarrow \infty$ for local trit-flip and local trit-phase-flip channels.

2. Initial states, noise model and geometric measure of quantum discord

The class of states with real parameters in a hybrid qubit-qutrit ($2 \otimes 3$) quantum system [31] are given by

$$\rho_{bc}(0) = a(|02\rangle\langle 02| + |12\rangle\langle 12|) + b(|\phi^+\rangle\langle \phi^+| + |\phi^-\rangle\langle \phi^-| + |\psi^+\rangle\langle \psi^+|) + c|\psi^-\rangle\langle \psi^-|, \quad (1)$$

where

$$\begin{aligned} |\phi^\pm\rangle &= \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), \\ |\psi^\pm\rangle &= \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle), \end{aligned} \quad (2)$$

and a , b , and c are three real parameters, and they satisfy the relation $2a + 3b + c = 1$. $|0\rangle$ and $|1\rangle$ are the two eigenstates of a two-level quantum system (qubit) or the eigenstates of a three-level quantum system (qutrit) with the other eigenstate $|2\rangle$. The two-parameter class of states $\rho_{bc}(0)$ can be obtained from an arbitrary state of a $2 \otimes 3$ quantum system by means of local quantum operations and classical communication (LOCC) [31].

Let us briefly recall the quantum discord of a bipartite state ρ in a Hilbert space $H^A \otimes H^B$. The quantum discord $\mathcal{D}(\rho^{AB})$, a measures of all quantum correlations, is defined as the difference between the total correlation and the classical correlation [11, 15], that is,

$$\mathcal{D}(\rho^{AB}) = I(\rho^{AB}) - C(\rho^{AB}), \quad (3)$$

where

$$I(\rho^{AB}) = S(\rho^A) + S(\rho^B) - S(\rho^{AB}). \quad (4)$$

Here, $I(\rho^{AB})$ represents the quantum mutual information (the total amount of correlations) of the two-subsystem state ρ^{AB} . $\rho^{A(B)} = \text{tr}_{B(A)}(\rho^{AB})$ is the reduced density matrix for the subsystem $A(B)$. $S(\rho) = -\text{tr}(\rho \log_2 \rho)$ is the von Neumann entropy of the system in the state ρ . The other quantity, $C(\rho^{AB})$, is interpreted [11, 32] as a measure of the classical correlation of the two subsystems AB in the state ρ^{AB} and it is defined as the maximal information that one can obtain, for example, about B by performing the complete measurement Π_k on H^A ,

$$C_B(\rho^{AB}) = \sup_{\Pi_k} \left\{ S(\rho^B) - \sum_k P_k S(\rho^{B|k}) \right\}, \quad (5)$$

where $\rho^{B|k} = \frac{1}{P_k}(\Pi_k \otimes I_B)\rho^{AB}(\Pi_k \otimes I_B)$ is the postmeasurement state of B after obtaining the outcome k on A with the probability $P_k = \text{tr}((\Pi_k \otimes I_B)\rho^{AB}(\Pi_k \otimes I_B))$. Π_k is a set of one-dimensional projectors on H^A .

We note that for a general mixed state, the (one-side) classical correlation of Eq.(5) may be asymmetric with respect to the choice of subsystem to be measured ($C_A(\rho) \neq C_B(\rho)$), that is, the quantum discord in Eq.(3) is not symmetric ($D_A(\rho) \neq D_B(\rho)$). However, it is known that $D_A(\rho), D_B(\rho) \geq 0$, and $D_A(\rho) = D_B(\rho) = 0$ if and only if ρ is a classical-quantum state, and ρ has the following expression:

$$\rho^{AB} = \sum_{i,j} P_{ij} |i\rangle_A \langle i| \otimes |j\rangle_B \langle j|, \quad (6)$$

where P_{ij} is a probability distribution, and $|i\rangle_A$ and $|j\rangle_B$ are the orthonormal bases of system A and B , respectively.

GMQD: the geometric measure of quantum discord of the state ρ is given by [17]

$$D_G(\rho) = \min_{\chi} \|\rho - \chi\|^2, \quad (7)$$

where the minimum is taken over the set of zero-discord states [i.e., $D(\chi) = 0$] and $\|\rho - \chi\|^2 = \text{tr}(\rho - \chi)^2$ is the square norm in the Hilbert-Schmidt space.

For any two-qubit state

$$\rho^{AB} = \frac{1}{4}[I \otimes I + \sum_{i=1}^3 (x_i \sigma_i \otimes I + y_i I \otimes \sigma_i) + \sum_{i,j=1}^3 (r_{ij} \sigma_i \otimes \sigma_j)], \quad (8)$$

its GMQD is given by

$$D_G(\rho) = \frac{1}{4}(\|X\|^2 + \|R\|^2 - k_{\max}), \quad (9)$$

where σ_i are the Pauli spin matrices, $X = (x_1, x_2, x_3)^T$, R is the matrix with elements r_{ij} , and k_{\max} is the maximal eigenvalue of the matrix $K = XX^T + RR^T$.

For a general bipartite system $H^A \otimes H^B$, if we choose orthonormal basis sets in local Hilbert-Schmidt spaces of Hermitian operators $\{X_i\}$ and $\{Y_j\}$ ($i = 1, \dots, d_A^2$ and $j = 1, \dots, d_B^2$), any bipartite state can be written as

$$\rho = \sum_{ij} c_{ij} X_i \otimes Y_j, \quad (10)$$

where $c_{ij} = \text{tr}(\rho X_i \otimes Y_j)$. Its GMQD can be expressed as [18]

$$D_G(\rho) = \text{tr}(CC^T) - \max_A \text{tr}(ACC^T A^T), \quad (11)$$

where $C = (c_{ij})$ and the maximum is taken over all $d_A \times d_A^2$ -dimensional matrices $A = (a_{ki})$. $a_{ki} = \text{tr}(|k\rangle\langle k|X_i)$. $\{|k\rangle\}$ is any orthonormal base for H^A . Here $k = 1, 2, \dots, d_A$ and $i = 1, 2, \dots, d_A^2$.

In our physical model of noise for a qubit-qutrit system (composed of a two-level subsystem A and a three-level subsystem B), the two subsystems interact with their environments independently. The evolved states of the initial density matrix of such a system when it is influenced by multi-local environments can be given compactly by

$$\rho_{bc}^{AB}(t) = \sum_{i=1}^2 \sum_{j=1}^3 F_j^B E_i^A \rho_{bc}^{AB}(0) E_i^{A\dagger} F_j^{B\dagger}. \quad (12)$$

Here, the operators E_i^A and F_j^B are the Kraus operators which are used to describe the noise channels acting on the qubit A and the qutrit B , respectively. They satisfy the completeness relations $\sum_i E_i^{A\dagger} E_i^A = I$ and $\sum_j F_j^{B\dagger} F_j^B = I$ for all t .

3. Geometric measure of quantum discord for qubit-qutrit systems

It is important to consider the behavior of quantum correlations under the action of decoherence. In this section, we investigate what happens to the quantum correlations (i.e., GMQD) in a qubit-qutrit system under common noise channels for qubit (qutrit): dephasing, phase-flip, bit-(trit-) flip, bit-(trit-) phase-flip, and depolarizing channels. The Kraus operators describing these channels for a single qubit A and a single qutrit B are presented in Appendix. The time-dependent parameters are defined as $\gamma_A = 1 - e^{-t\Gamma_A}$ and $\gamma_B = 1 - e^{-t\Gamma_B}$, $\gamma_A, \gamma_B \in [0, 1]$ and Γ_A (Γ_B) denotes the decay rate of the subsystem A (B). The two specific environment noise situations will be considered: (i) local and (ii) multi-local. In the case (i), only one part of a qubit-qutrit system (S) interacts with its environment. In the case (ii), both the two parts of S interact with their local environments, independently.

We choose 4 Hermitian matrices for each H^A ,

$$X_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad X_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad X_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad X_4 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (13)$$

and 9 matrices Hermitian matrices for each H^B ,

$$\begin{aligned} Y_1 &= \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, & Y_2 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, & Y_3 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ Y_4 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, & Y_5 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, & Y_6 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \\ Y_7 &= \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, & Y_8 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, & Y_9 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}. \end{aligned} \quad (14)$$

Any orthogonal base for H^A can written as

$$\begin{aligned} |\theta_{||}\rangle &= \cos \theta |0\rangle + e^{i\varphi} \sin \theta |1\rangle, \\ |\theta_{\perp}\rangle &= \sin \theta |0\rangle - e^{i\varphi} \cos \theta |1\rangle. \end{aligned} \quad (15)$$

The matrix $A = (a_{ki})$ in Eq.(11) with the matrix elements: $a_{ki} = \text{tr}(|k\rangle\langle k|X_i)$. Here, $\{|k\rangle\}$ is any orthonormal base for H^A shown in Eq.(15), and X_i is a set of Hermitian matrices for H^A shown in Eq.(13). The matrix can be presented as

$$A = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & \sin 2\theta \cos \varphi & \sin 2\theta \sin \varphi & \cos 2\theta \\ 1 & -\sin 2\theta \cos \varphi & -\sin 2\theta \sin \varphi & -\cos 2\theta \end{pmatrix}. \quad (16)$$

(1) *Channel without noise.* The coefficient elements $c_{ij} = \text{tr}(\rho X_i \otimes Y_j)$ of the matrix C in Eq.(11) are given by

$$\begin{aligned} c_{11} &= \frac{1}{\sqrt{6}}, \\ c_{17} &= -\frac{2 - 9b - 3c}{2\sqrt{3}}, \\ c_{22} &= c_{33} = c_{44} = \frac{1}{2}(b - c), \end{aligned} \quad (17)$$

and all the remaining matrix elements are zero.

By replacing the factors A and $C = (c_{ij})$ in Eq.(11) with Eqs.(16) and (17), respectively (that is, D_G is the function of θ and φ), and calculating the minimum of D_G over θ and φ , we obtain the GMQD for the qubit-qutrit systems under a channel without noise, that is

$$D_G(\rho^{AB}) = \frac{1}{2}(b - c)^2. \quad (18)$$

(2) *Multi-local dephasing channel.* The coefficient matrix elements c_{ij} are given by

$$\begin{aligned} c_{11} &= \frac{1}{\sqrt{6}}, \\ c_{17} &= -\frac{1}{2\sqrt{3}}(2 - 9b - 3c), \\ c_{22} &= c_{33} = \frac{1}{2}(b - c)\sqrt{(1 - \gamma_A)(1 - \gamma_B)}, \\ c_{44} &= \frac{1}{2}(b - c), \end{aligned} \quad (19)$$

and all the remaining matrix elements are zero.

By replacing the factors A and $C = (c_{ij})$ in Eq.(11) with Eqs.(16) and (19), respectively, and calculating the minimum of D_G over θ and φ , we obtain the GMQD for the qubit-qutrit systems under a multi-local dephasing channel, that is

$$D_G(\rho^{AB})_1 = \frac{1}{2}(b - c)^2(1 - \gamma_A)(1 - \gamma_B). \quad (20)$$

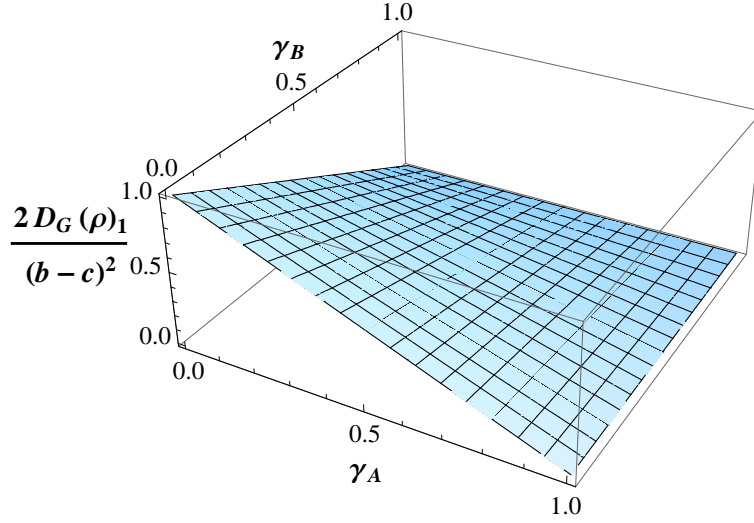


FIG. 1: Dynamics of GMQD for the system undergoing the multi-local dephasing noise. γ is the time-dependent parameter and $\gamma_A = 1 - e^{-t\Gamma_A}$, $\gamma_B = 1 - e^{-t\Gamma_B}$.

Its dynamics is shown in Fig.1. In the paper, we consider the parameters $c \neq b$, that is, the initial state is a quantum correlation state.

(3) *Multi-local phase-flip channel.* The coefficient matrix elements c_{ij} are given by

$$\begin{aligned} c_{11} &= \frac{1}{\sqrt{6}}, \\ c_{17} &= -\frac{1}{2\sqrt{3}}(2 - 9b - 3c), \\ c_{22} &= c_{33} = \frac{1}{2}(b - c)(1 - \gamma_A)(1 - \gamma_B), \\ c_{44} &= \frac{1}{2}(b - c), \end{aligned} \tag{21}$$

and all the remaining matrix elements are zero.

By replacing the factors A and $C = (c_{ij})$ in Eq.(11) with Eqs.(16) and (21), respectively, and calculating the minimum of D_G over θ and φ , we obtain the GMQD for the system, that is

$$D_G(\rho^{AB})_2 = \frac{1}{2}(b - c)^2(1 - \gamma_A)^2(1 - \gamma_B)^2. \tag{22}$$

Its dynamics is shown in Fig.2.

(4) *Multi-local bit- (trit-) flip channel.* The coefficient matrix elements c_{ij} are given by

$$\begin{aligned} c_{11} &= \frac{1}{\sqrt{6}}, \\ c_{17} &= \frac{1}{2\sqrt{3}}(2 - 9b - 3c)(\gamma_B - 1), \\ c_{22} &= -\frac{1}{6}(b - c)(2\gamma_B - 3), \\ c_{25} &= c_{28} = \frac{1}{6}(b - c)\gamma_B, \\ c_{33} &= \frac{1}{6}(b - c)(2\gamma_B - 3)(\gamma_A - 1), \\ c_{36} &= -c_{39} = \frac{1}{6}(b - c)(\gamma_A - 1)\gamma_B, \\ c_{44} &= \frac{1}{2}(b - c)(\gamma_B - 1)(\gamma_A - 1), \end{aligned} \tag{23}$$

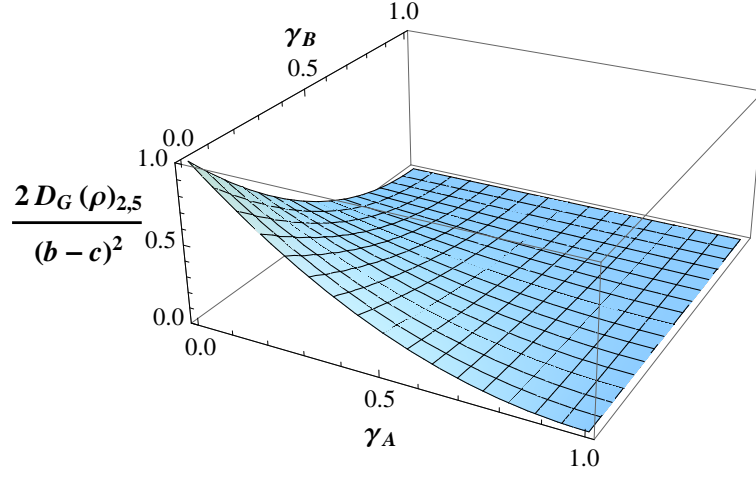


FIG. 2: Dynamics of GMQD for the system undergoing the multi-local phase-flip and depolarizing noises.

and all the remaining matrix elements are zero.

By replacing the factors A and $C = (c_{ij})$ in Eq.(11) with Eqs.(16) and (23), respectively, and calculating the minimum of D_G over θ and φ , we obtain the GMQD for the system, that is

$$D_G(\rho^{AB})_3 = \frac{1}{12}(b-c)^2(1-\gamma_A)^2(6+5(\gamma_B-2)\gamma_B). \quad (24)$$

Its dynamics is shown in Fig.3.

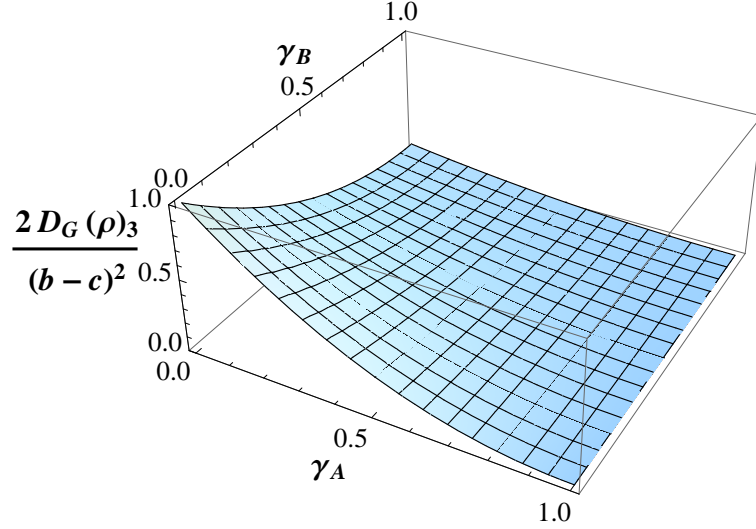


FIG. 3: Dynamics of GMQD for the system undergoing the multi-local bit- (trit-) flip noise.

(5) *Multi-local bit- (trit-) phase-flip channel.* The coefficient matrix elements c_{ij} are given by

$$\begin{aligned}
c_{11} &= \frac{1}{\sqrt{6}}, \\
c_{17} &= \frac{1}{2\sqrt{3}}(2 - 9b - 3c)(\gamma_B - 1), \\
c_{22} &= \frac{1}{6}(b - c)(2\gamma_B - 3)(\gamma_A - 1), \\
c_{25} &= c_{28} = \frac{1}{12}(b - c)(\gamma_A - 1)\gamma_B, \\
c_{33} &= -\frac{1}{6}(b - c)(2\gamma_B - 3), \\
c_{36} &= -c_{39} = \frac{1}{12}(b - c)\gamma_B, \\
c_{44} &= \frac{1}{2}(b - c)(\gamma_B - 1)(\gamma_A - 1),
\end{aligned} \tag{25}$$

and all the remaining matrix elements are zero.

By replacing the factors A and $C = (c_{ij})$ in Eq.(11) with Eqs.(16) and (25), respectively, and calculating the minimum of D_G over θ and φ , we obtain the GMQD for the system, that is

$$D_G(\rho^{AB})_4 = \frac{1}{24}(b - c)^2(1 - \gamma_A)^2(12 + \gamma_B(9\gamma_B - 20)). \tag{26}$$

Its dynamics is shown in Fig.4.

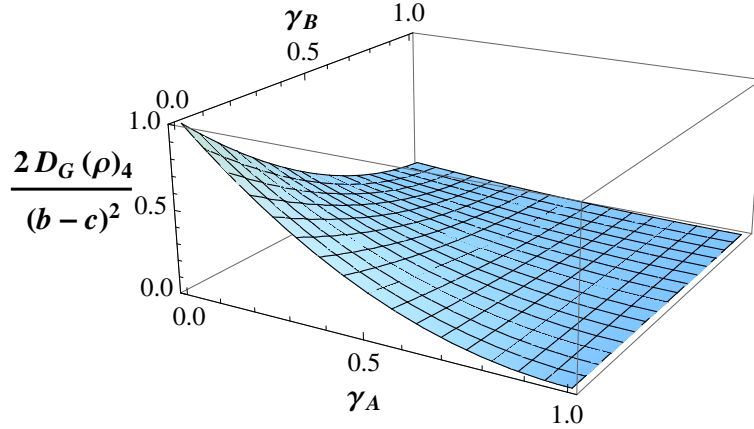


FIG. 4: Dynamics of GMQD for the system undergoing the multi-local bit- (trit-) phase-flip noise.

(6) *Multi-local depolarizing channel.* The coefficient matrix elements c_{ij} are given by

$$\begin{aligned}
c_{11} &= \frac{1}{\sqrt{6}}, \\
c_{17} &= \frac{1}{2\sqrt{3}}(2 - 9b - 3c)(\gamma_B - 1), \\
c_{22} &= c_{33} = c_{44} = \frac{1}{2}(b - c)(1 - \gamma_A)(1 - \gamma_B).
\end{aligned} \tag{27}$$

all the remaining matrix elements are zero.

By replacing the factors A and $C = (c_{ij})$ in Eq.(11) with Eqs.(16) and (27), respectively, and calculating the minimum of D_G over θ and φ , we obtain the GMQD for the system, that is

$$D_G(\rho^{AB})_5 = \frac{1}{2}(b - c)^2(1 - \gamma_A)^2(1 - \gamma_B)^2. \tag{28}$$

Its dynamics is shown in Fig.2.

(7) *Local qubit noise only.* Consider $\gamma_B = 0$, the GMQD can be calculated as

$$\begin{aligned} D_G^{(1)}(\rho^{AB})_6 &= \frac{1}{2}(b-c)^2(1-\gamma_A), \\ D_G^{(2)}(\rho^{AB})_6 &= \frac{1}{2}(b-c)^2(1-\gamma_A)^2. \end{aligned} \quad (29)$$

Here $D_G^{(1)}(\rho^{AB})_6$ corresponds to dephasing channel, and $D_G^{(2)}(\rho^{AB})_6$ corresponds to phase-flip, bit-flip, bit-phase-flip or depolarizing channels. The dynamics of GMQD for these cases are shown in Fig.5

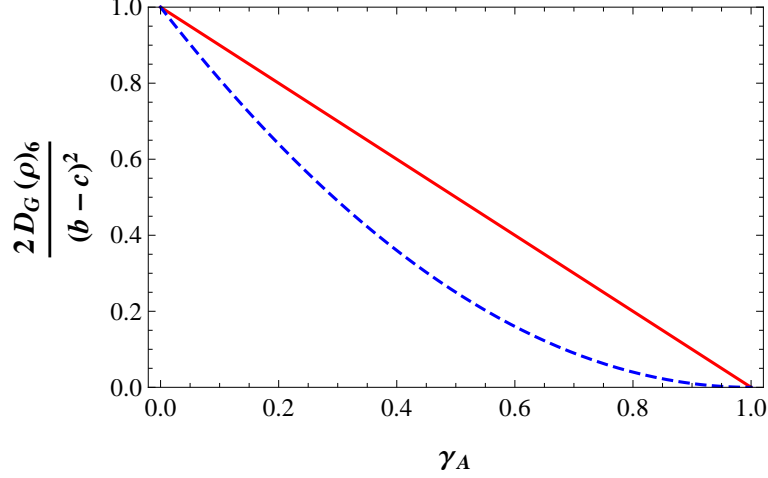


FIG. 5: Dynamics of GMQD for the system undergoing the various local noises which act on the qubit alone. The solid and short-dashed lines correspond to dephasing and phase-flip (or bit-flip, bit-phase-flip, depolarizing) noises, respectively.

(8) *Local qutrit noise only.* Consider $\gamma_A = 0$, the GMQD can be calculated as

$$\begin{aligned} D_G^{(1)}(\rho^{AB})_7 &= \frac{1}{2}(b-c)^2(1-\gamma_B), \\ D_G^{(2)}(\rho^{AB})_7 &= \frac{1}{2}(b-c)^2(1-\gamma_B)^2, \\ D_G^{(3)}(\rho^{AB})_7 &= \frac{1}{12}(b-c)^2(6+5(\gamma_B-2)\gamma_B), \\ D_G^{(4)}(\rho^{AB})_7 &= \frac{1}{24}(b-c)^2(12+\gamma_B(9\gamma_B-20)). \end{aligned} \quad (30)$$

Here $D_G^{(1)}(\rho^{AB})_7$, $D_G^{(2)}(\rho^{AB})_7$, $D_G^{(3)}(\rho^{AB})_7$, and $D_G^{(4)}(\rho^{AB})_7$ corresponds to dephasing, phase-flip (or depolarizing), trit-flip, and trit-phase-flip channels, respectively. The dynamics of GMQD for these cases are shown in Fig.6.

Eqs.(29) and (30) show that the environment, which acts on the qubit alone and causes phase-flip, bit-flip, bit-phase-flip, and depolarizing, affects the GMQD of a hybrid qubit-qutrit system in the same way. However, if the environment acts on the qutrit alone, only the phase-flip and depolarizing channels affect the GMQD of the qubit-qutrit system in the same way.

4. Discussion and Conclusion

Quantum discord \mathcal{D} for a two-parameter class of states in a hybrid qubit-qutrit system was discussed by Ali [16] in 2010. We have studied GMQD and its dynamics under the influence of various dissipative channels, including local and multi-local dephasing, phase-flip, bit- (trit-) flip, bit- (trit-) phase-flip, and depolarizing channels, in the first time. Moreover, the explicit analytical expressions were gave out. Our results show that environment, which causes dephasing, phase-flip, bit- (trit-) flip, bit- (trit-) phase-flip, and depolarizing of a qubit-qutrit system, affects the quantum correlations of a hybrid qubit-qutrit system in very different ways. All these dynamic evolutions do

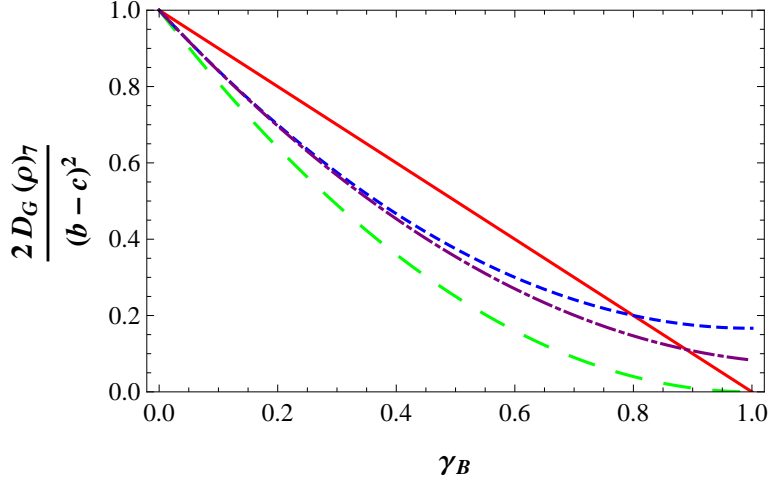


FIG. 6: Dynamics of GMQD for the system undergoing the various local noises which act on the qutrit alone. The solid, dashed, short-dashed and dashed-dotted lines correspond to dephasing, phase-flip (or depolarizing), trit-flip, and trit-phase-flip noises, respectively.

not lead to a sudden vanishing of GMQD. Quantum correlations vanish at asymptotic time for local or multi-local dephasing, phase-flip, and depolarizing noise channels, while it cannot be destroyed completely even though $t \rightarrow \infty$ for local trit-flip and local trit-phase-flip channels. The states shown in Eq.(1) with $a = 0$ (that is, $c = 1 - 3b$) are equivalent to Werner states [35] in $2 \otimes 2$ system with different noise channels. Compared with the states in two-qubit systems, their quantum correlation all vanishes at a asymptotic time and can not occurs quantum correlation sudden death and sudden birth under an arbitrary Markovian dynamics [19, 21, 22]. This phenomenon maybe provide some important information for the application of GMQD in hybrid qubit-qutrit systems in quantum information.

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Appendix A: Various dissipative channels

Dephasing channels: the set of Kraus operators for a single qubit A that reproduces the effect of a dephasing channel are given by [33]

$$E_1^A = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma_A} \end{pmatrix} \otimes I_3, \quad E_2^A = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{\gamma_A} \end{pmatrix} \otimes I_3, \quad (\text{A1})$$

and those for a single qutrit B can be written as [25]

$$F_1^B = I_2 \otimes \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{1-\gamma_B} & 0 \\ 0 & 0 & \sqrt{1-\gamma_B} \end{pmatrix}, \quad F_2^B = I_2 \otimes \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sqrt{\gamma_B} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad F_3^B = I_2 \otimes \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sqrt{\gamma_B} \end{pmatrix}. \quad (\text{A2})$$

The time-dependent parameters are defined as $\gamma_A = 1 - e^{-t\Gamma_A}$ and $\gamma_B = 1 - e^{-t\Gamma_B}$. Here $\gamma_A, \gamma_B \in [0, 1]$. Γ_A (Γ_B) denotes the decay rate of the subsystem A (B).

Phase-flip channels: the Kraus operators describing the phase-flip channel for a single qubit A are given by [33]

$$E_1^A = \sqrt{1 - \frac{\gamma_A}{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes I_3, \quad E_2^A = \sqrt{\frac{\gamma_A}{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes I_3, \quad (\text{A3})$$

and those for a single qutrit B can be written as

$$F_1^B = I_2 \otimes \sqrt{1 - \frac{2\gamma_B}{3}} I_3, \quad F_2^B = I_2 \otimes \sqrt{\frac{\gamma_B}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i2\pi/3} & 0 \\ 0 & 0 & e^{i2\pi/3} \end{pmatrix}, \quad F_3^B = I_2 \otimes \sqrt{\frac{\gamma_B}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i2\pi/3} & 0 \\ 0 & 0 & e^{-i2\pi/3} \end{pmatrix}, \quad (\text{A4})$$

where $\gamma_A = 1 - e^{-t\Gamma_A}$, $\gamma_B = 1 - e^{-t\Gamma_B}$, and $\gamma_A, \gamma_B \in [0, 1]$. Γ_A (Γ_B) represents the decay rate of the subsystem A (B).

Bit-(Trit-) flip channels: the Kraus operators describing the bit-flip channel for a single qubit A are given by [33]

$$E_1^A = \sqrt{1 - \frac{\gamma_A}{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes I_3, \quad E_2^A = \sqrt{\frac{\gamma_A}{2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes I_3, \quad (\text{A5})$$

and those for a single qutrit B can be written as

$$F_1^B = I_2 \otimes \sqrt{1 - \frac{2\gamma_B}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad F_2^B = I_2 \otimes \sqrt{\frac{\gamma_B}{3}} \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad F_3^B = I_2 \otimes \sqrt{\frac{\gamma_B}{3}} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad (\text{A6})$$

where $\gamma_A = 1 - e^{-t\Gamma_A}$, $\gamma_B = 1 - e^{-t\Gamma_B}$, and $\gamma_A, \gamma_B \in [0, 1]$.

Bit-(Trit-) phase-flip channels: the Kraus operators describing the bit-phase flip channel for a single qubit A are given by [33]

$$E_1^A = \sqrt{1 - \frac{\gamma_A}{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes I_3, \quad E_2^A = \sqrt{\frac{\gamma_A}{2}} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \otimes I_3, \quad (\text{A7})$$

and those for a single qutrit B can be written as

$$\begin{aligned} F_1^B &= I_2 \otimes \sqrt{1 - \frac{2\gamma_B}{3}} I_3, & F_2^B &= I_2 \otimes \sqrt{\frac{\gamma_B}{6}} \begin{pmatrix} 0 & 0 & e^{i2\pi/3} \\ 1 & 0 & 0 \\ 0 & e^{-i2\pi/3} & 0 \end{pmatrix}, \\ F_3^B &= I_2 \otimes \sqrt{\frac{\gamma_B}{6}} \begin{pmatrix} 0 & 0 & e^{-i2\pi/3} \\ 1 & 0 & 0 \\ 0 & e^{i2\pi/3} & 0 \end{pmatrix}, & F_4^B &= I_2 \otimes \sqrt{\frac{\gamma_B}{6}} \begin{pmatrix} 0 & e^{-i2\pi/3} & 0 \\ 0 & 0 & e^{i2\pi/3} \\ 1 & 0 & 0 \end{pmatrix}, \\ F_5^B &= I_2 \otimes \sqrt{\frac{\gamma_B}{6}} \begin{pmatrix} 0 & e^{i2\pi/3} & 0 \\ 0 & 0 & e^{-i2\pi/3} \\ 1 & 0 & 0 \end{pmatrix}, & & \end{aligned} \quad (\text{A8})$$

where $\gamma_A = 1 - e^{-t\Gamma_A}$, $\gamma_B = 1 - e^{-t\Gamma_B}$, and $\gamma_A, \gamma_B \in [0, 1]$.

Depolarizing channels: the set of Kraus operators that reproduces the effect of the depolarizing channel for a single qubit A are given by [33]

$$E_1^A = \sqrt{1 - \frac{3\gamma_A}{4}} I_6, \quad E_2^A = \sqrt{\frac{\gamma_A}{4}} \sigma_1 \otimes I_3, \quad E_3^A = \sqrt{\frac{\gamma_A}{4}} \sigma_2 \otimes I_3, \quad E_4^A = \sqrt{\frac{\gamma_A}{4}} \sigma_3 \otimes I_3, \quad (\text{A9})$$

where σ_i ($i = 1, 2, 3$) are the three Pauli matrices. The Kraus operators describing a single-qutrit depolarizing noise are given by [?]

$$\begin{aligned} F_1^B &= I_2 \otimes \sqrt{1 - \frac{8\gamma_B}{9}} I_3, & F_2^B &= I_2 \otimes \frac{\sqrt{\gamma_B}}{3} Y, & F_3^B &= I_2 \otimes \frac{\sqrt{\gamma_B}}{3} Z, \\ F_4^B &= I_2 \otimes \frac{\sqrt{\gamma_B}}{3} Y^2, & F_5^B &= I_2 \otimes \frac{\sqrt{\gamma_B}}{3} YZ, & F_6^B &= I_2 \otimes \frac{\sqrt{\gamma_B}}{3} Y^2 Z, \\ F_7^B &= I_2 \otimes \frac{\sqrt{\gamma_B}}{3} YZ^2, & F_8^B &= I_2 \otimes \frac{\sqrt{\gamma_B}}{3} Y^2 Z^2, & F_9^B &= I_2 \otimes \frac{\sqrt{\gamma_B}}{3} Z^2, \end{aligned} \quad (\text{A10})$$

where

$$Y = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i2\pi/3} & 0 \\ 0 & 0 & e^{-i2\pi/3} \end{pmatrix}, \quad (\text{A11})$$

and $\gamma_A = 1 - e^{-t\Gamma_A}$, $\gamma_B = 1 - e^{-t\Gamma_B}$, $\gamma_A, \gamma_B \in [0, 1]$.

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- [1] Braunstein S.L., Caves C.M., Jozsa R., Linden N., Popescu S., Schack R.: Separability of very noisy mixed states and implications for NMR quantum computing. *Phys. Rev. Lett.* **83**, 1054 (1999)
 - [2] Meyer D.A.: Sophisticated Quantum Search Without Entanglement. *Phys. Rev. Lett.* **85**, 2014 (2000)
 - [3] Datta A., Flammia S.T., Caves C.M.: Entanglement and the power of one qubit. *Phys. Rev. A* **72**, 042316 (2005)
 - [4] Datta A., Vidal G.: Role of entanglement and correlations in mixed-state quantum computation. *Phys. Rev. A* **75**, 042310 (2007)
 - [5] Datta A., Shaji A., Caves C.M.: Quantum discord and the power of one qubit. *Phys. Rev. Lett.* **100**, 050502 (2008)
 - [6] Lanyon B.P., Barbieri M., Almeida M.P., White A.G.: Experimental quantum computing without entanglement. *Phys. Rev. Lett.* **101**, 200501 (2008)
 - [7] Roa L., Retamal J.C., Alid-Vaccarezza M.: Dissonance is required for assisted optimal state discrimination. *Phys. Rev. Lett.* **107**, 080401 (2011)
 - [8] Roa L., Olivares-Rentería G.A.: Decoherence assisting a measurement-driven quantum evolution process. *Phys. Rev. A* **73**, 062327 (2006)
 - [9] López C.E., Romero G., Retamal J.C.: Dynamics of entanglement transfer through multipartite dissipative systems. *Phys. Rev. A* **81**, 062114 (2010)
 - [10] López C.E., Romero G., Lastra F., Solano E., Retamal J.C.: sudden birth versus sudden death of entanglement in multipartite systems. *Phys. Rev. Lett.* **101**, 080503 (2008)
 - [11] Ollivier H., Zurek W.H.: Quantum discord: a measure of the quantumness of correlations. *Phys. Rev. Lett.* **88**, 017901 (2001)
 - [12] Fanchini F.F., Werlang T., Brasil C.A., Arruda L.G.E., Caldeira A.O.: Non-Markovian dynamics of quantum discord. *Phys. Rev. A* **81**, 052107 (2010)
 - [13] Sarandy M.S.: Classical correlation and quantum discord in critical systems, *Phys. Rev. A* **80**, 022108 (2009)
 - [14] Ali M., Rau A.R.P., Alber G.: Quantum discord for two-qubit X states. *Phys. Rev. A* **81**, 042105 (2010)
 - [15] Luo S.L.: Quantum discord for two-qubit systems. *Phys. Rev. A* **77**, 042303 (2008)
 - [16] Ali M.: Quantum discord for a two-parameter class of states in $2 \otimes d$ quantum systems. *J. Phys. A Math. Theor.* **43**, 495303 (2010)
 - [17] Dakić B., Vedral V., Brukner Č.: Necessary and sufficient condition for nonzero quantum discord. *Phys. Rev. Lett.* **105**, 190502 (2010)
 - [18] Luo S.L., Fu S.S.: Geometric measure of quantum discord. *Phys. Rev. A* **82**, 034302 (2010)
 - [19] Werlang T., Souza S., Fanchini F.F., Villas Boas C.J.: Robustness of quantum discord to sudden death. *Phys. Rev. A* **80**, 024103 (2009)
 - [20] Maziero J., Céleri L.C., Serra R.M., Vedral V.: Classical and quantum correlations under decoherence. *Phys. Rev. A* **80**, 044102 (2009)
 - [21] Maziero J., Werlang T., Fanchini F.F., Céleri L.C., Serra R.M.: System-reservoir dynamics of quantum and classical correlations. *Phys. Rev. A* **81**, 022116 (2010)
 - [22] Ferraro A., Aolita L., Cavalcanti D., Cucchietti F.M., Acín A.: Almost all quantum states have non-classical correlations. *Phys. Rev. A* **81**, 052318 (2010)
 - [23] Yu T., Eberly J.H.: The end of an entanglement. *Science* **316**, 555 (2007)
 - [24] Yu T., Eberly J.H.: Sudden death of entanglement. *Science* **323**, 598 (2009)
 - [25] Ann K. and Jaeger G.: Entanglement sudden death in qubit-qutrit systems. *Phys. Lett. A* **372**, 579 (2008)
 - [26] Khan S.: Entanglement sudden death in qubit-qutrit systems under depolarizing noise. arXiv: quant-ph/1012.1028.
 - [27] Ali M., Rau A.R.P., Ranade K.: Disentanglement in qubit-qutrit systems. arXiv:quant-ph/0710.2238
 - [28] Ali M., Phil M.: Quantum control of finite-time disentanglement in qubit-qubit and qubit-qutrit systems. Darmstädter Dissertation, Darmstadt (2009)
 - [29] Ramzan M., Khan M.K.: Decoherence and entanglement degradation of a qubit-qutrit system in non-inertial frames. *Quant. Inf. Process.* **11**, 443 (2012)
 - [30] Karpat G., Gedik Z.: Correlation dynamics of qubit-qutrit systems in a classical dephasing environment. *Phys. Lett. A* **375**, 4166 (2011)
 - [31] Chi D.P., Lee S.: Entanglement for a two-parameter class of states in $2 \otimes n$ quantum system. *J. Phys. A: Math. Gen.* **36**, 11503 (2003)
 - [32] Henderson L., Vedral V.: Classical, quantum and total correlations. *J. Phys. A: Math. Gen.* **34**, 6899 (2001)
 - [33] Nielsen M. A. and Chuang I. L.: *Quantum Computation and Quantum Information*. Cambridge University Press, Cambridge, UK, (2000)
 - [34] Ann K., Jaeger G.: Entanglement sudden death in qubit-qutrit systems. *Phys. Lett. A* **372**, 579 (2008)
 - [35] Werner R.F.: Quantum states with Einstein-Podolsky-Rosen correlations admitting a hidden-variable model. *Phys. Rev. A* **40**, 4277 (1989)